PROBLEM BASED ON

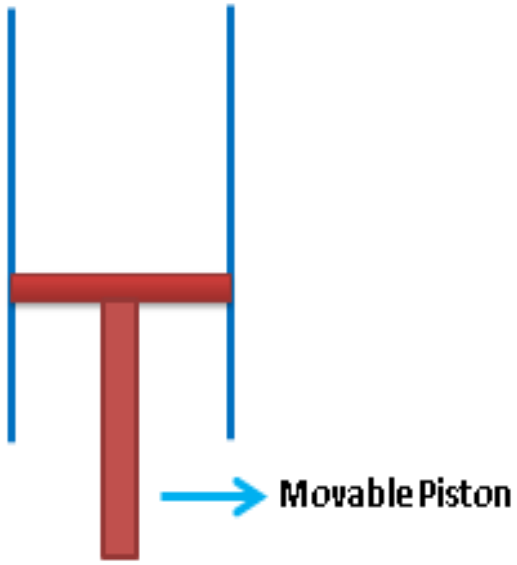
Modes of Vibration

QUESTION:

A metre long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz), when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. Ignore edge effect.

Solution:

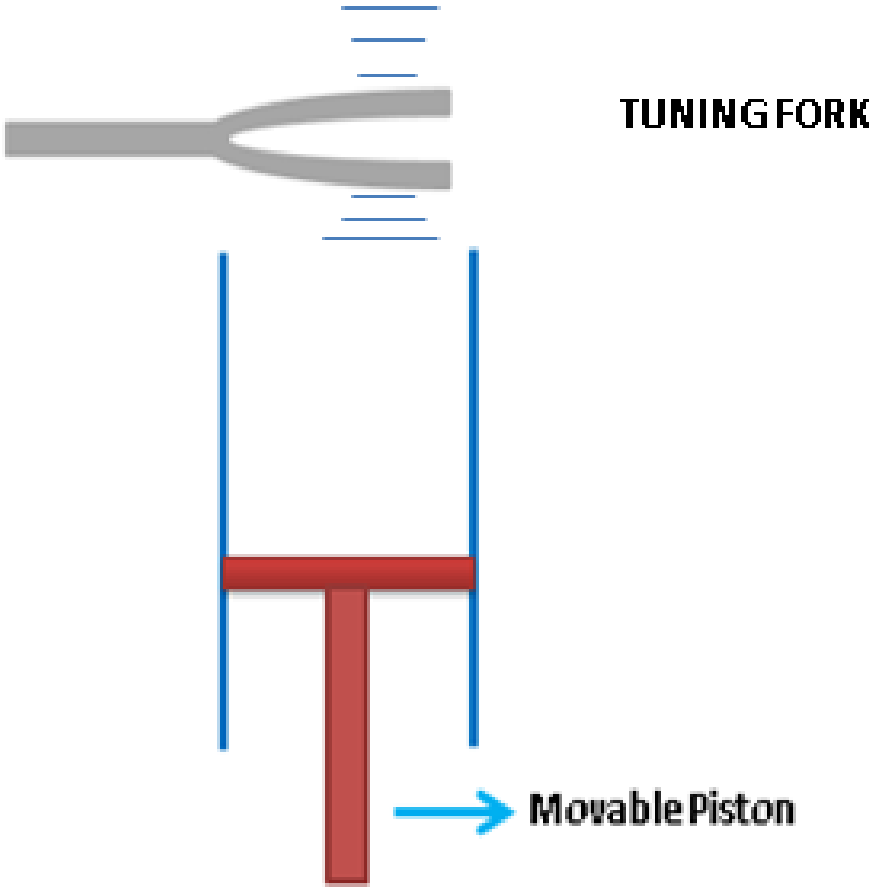
Since the tube is attached with a movable piston at one end, it will behave as pipe which is closed at one end and open at other end.



A closed tube produces odd harmonics only and even harmonics are absent in a closed tube.

The frequency of the nth mode of vibration for a closed end pipe is given by

$$\nu_n = \frac{(2n-1)v}{4L} \qquad \text{..... (1)}$$


TUNING FORK
Movable Piston

Here tube is in resonance with the tuning fork so that tube vibrates with the same natural frequency as tuning fork.

In Resonance, the frequency of the external force is equal to the natural frequency of the body.

Suppose the n_1th mode of vibration is produced, when the length of the tube (Open at one end) is 25.5 cm.

As it is in resonance with the tuning fork of frequency 340 Hz,

frequency of tuning fork is equal to frequency of nth mode of vibration of closed end

$$340 = \frac{(2n_1 - 1)v}{4 \times 25.5} \quad \text{..... (2)}$$

When the tube length is increased to 79.3 cm, the next i.e. (n_1+1)th mode of vibration comes in resonance with the same tuning fork.

Therefore,

From above equation (1), we have

$$340 = \frac{2[(n_1+1)-1]v}{4\times 79.3} \qquad \text{..... (3)}$$

From equation (2) and (3), we have

$$\frac{(2n_1 - 1)v}{4 \times 25.5} = \frac{2[(n_1 + 1) - 1]v}{4 \times 79.3}$$

$$\frac{(2n_1 - 1)}{25.5} = \frac{[2n_1 + 1]}{79.3}$$

$$2n_1 \times 79.3 = 2 \times 25.5n_1 + 25.5$$

$$107.6\, n_1 = 104.8$$

$$n_1 \approx 1$$

By substituting the value of n_1 in equation (2), we have

$$340 = \frac{(2 \times 1 - 1)v}{4 \times 25.5}$$

$$v = 34,680\ cm\ s^{-1}$$

$$= 346.8\ m\ s^{-1}$$

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